

The Emergence of NURBS by Robin Forrest

I have been fortunate to participate in some significant developments in computer graphics. Elsewhere,⁵ I have described the origins of the Bernstein form of the Bézier curve, the form that is now universal, and how that led to the use of B-splines for computer-aided geometric design (CAGD). A second thread is the emergence of nonuniform rational B-splines (NURBS) as the standard curve form for CAGD. NURBS followed logically Rich Riesenfeld's work at Syracuse University,⁸ K.J. Versprille's thesis on rational splines,¹¹ and L.C. Knapp's thesis on nonuniform splines.⁶

As a graduate student in the Mathematical Laboratory at Cambridge University, I was urged to seek help on curves and surfaces from in-house mathematicians: H.P.F. Swinnerton-Dyer admitted I had a problem, but said that such geometry had been in vogue at the turn of the century but was no longer of interest, and J.C.P. Miller, the noted compiler of mathematical tables, suggested that I would find all the answers in the manual for the 1936 Brunsviga calculator. Help of a totally different nature came when I spent a summer at the Massachusetts Institute of Technology working with Steven Coons.

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When I arrived in the other Cambridge, I was somewhat surprised to find that Coons was on sabbatical leave, but as this was just up the road at Harvard University, I was reassured. Coons was working with Ivan Sutherland, who had gathered Bob Sproull, Danny Cohen, and Ted Lee to work with him. Sutherland used to tease me about being at the technical school down the road, until I pointed out that all his degrees were from technical schools (Carnegie Tech, the California Institute of Technology, and MIT). Coons had arranged for me to have an office in Project MAC next to Doug Ross, who led the AED Group, but my status was enigmatic, visiting research students being an unknown category. Coons solved my problem by persuading the authorities to declare me a visiting research fellow, hence boosting my résumé.

In the summer of 1967, Coons was working on the famous "little red book,"¹ (*MAC-TR-41*), which despite its June date did not appear until the fall. L.G. Roberts, following his introduction of homogeneous coordinates to computer graphics,⁹ had demonstrated that conic arcs could be represented as rational parametric quadratic functions. At Boeing, M.S. Rowin had developed the T-conic,¹⁰ which merged the parametric cubic curves of J.C. Ferguson² with the conic arcs then conventionally used in aircraft lofting. T-conics were rational parametric cubics with a quadratic denominator that enabled the homogeneous component to be controlled by a single shape factor related to the conic shape factor. This neatly avoided problems with asymptotes but reduced the generality of the curve. Coons and I spent the

summer of 1967 developing the rational cubic form. My particular concern was to discover how best to specify the homogeneous coefficients that gave extra degrees of freedom in controlling curve shape. Thus, my 1968 PhD thesis³ contains recipes for defining straight lines and circular, elliptic, parabolic, and hyperbolic arcs in rational cubic terms; explores means for reparameterizing rational cubics while retaining the same shape, splitting curves for subsequent refinement; and discusses the use of rational blending functions to control patch interior shape.

Lee's Harvard thesis⁷ dealt with rational bicubic surface patches. Further work on controlling the shape of a rational cubic by specifying the intersection of the curve with a plane defined by the tangent points and the mid-chord appeared as a Cambridge CAD Group document in 1970.⁴ Coons moved to Syracuse in 1969 and sent Riesenfeld to work with me at Cambridge as a visiting research student in the summer of 1970 and Knapp in the summer of 1972. As a visiting professor at Syracuse, I was responsible with Coons for the supervision of Riesenfeld's thesis research on B-splines during 1971–1972.

A characteristic of much of the early work in computer graphics and CAGD was the mode of publication. With few exceptions, work was reported as theses, technical reports, and company internal documents rather than in journals and at conferences. The relatively small community working in the area managed to communicate via word of mouth and by exchanging papers thought to be of mutual interest. The Report Library that I started at Cambridge University soon became widely known, and, in some cases, others used it as a repository of material for record. It is a source of regret that much of the material gathered in the 1960s and early 1970s has never been published in conventional form.

References

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